① $\lim_{x \to 2} (3x-2) = 4$

Recall: From definition of limits, $|f(x) - L| < \varepsilon$ where L is the solution to the limit.

$\Rightarrow |f(x) - 4| < \varepsilon$ (Answer).

② $y = 4x^2 \tan x$

$u = 4x^2$ $\quad u' = \frac{du}{dx} = 8x$

$v = \tan x$ $\quad v' = \sec^2 x$

$uv' + vu'$

$\Rightarrow 4x^2(\sec^2 x) + \tan x(8x)$

$4x^2 \sec^2 x + 8x \tan x$ or $4x(x\sec^2 x + 2\tan x)$

③ $0 < |x-a| < \delta$ $x \to a$ (from definition of limits, $|x-a| < \delta$)

④ $x^{-3} + \frac{1}{x^7} = h(x)$

rearranging $\Rightarrow y = x^{-3} + x^{-7}$

$y' = -3x^{-4} + (-7x^{-8})$

$= -3x^{-4} - 7x^{-8}$ or $\frac{-3}{x^4} - \frac{7}{x^8}$ or $-\left(\frac{3}{x^4} + \frac{7}{x^8}\right)$

⑤ $\int \frac{1}{x^3}\,dx \Rightarrow \int x^{-3}\,dx$

$= \frac{x^{-3+1}}{-3+1} + c \Rightarrow \frac{x^{-2}}{-2} + c$ or $\frac{1}{-2x^2} + c$

⑥ $f(x) = c \Rightarrow \frac{dy}{dx} = 0$.

⑧ (0, 0) if $x^3y^3 - y = x$

$$x^3 \cdot (3y^2 \cdot y') + y^3 \cdot 3x^2 - y' = 1$$

$$x^3 3y^2 y' - y' = 1 - 3x^2y^3$$

$$\frac{y'\cancel{(x^3 3y^2 - 1)}}{\cancel{(x^3 3y^2 - 1)}} = \frac{1 - 3x^2y^3}{(x^3 3y^2 - 1)}$$

$$y' = \frac{dy}{dx} = \frac{1 - 3x^2y^3}{(x^3 3y^2 - 1)}$$

at (0, 0)

$$y' = \frac{1 - 3(0^2)(0^3)}{(0^3 3(0^2) - 1)} = \frac{1}{-1} = -1$$

⑨ Odd function implies that $f(-x) \neq f(x)$

* $\sin x$
* $\cos x$
* $\operatorname{cosec} x$

taking 30° as x

* $\sin(30) = 0.5$ and $\sin(-30) = -0.5$
* $\operatorname{cosec}(30) = \frac{1}{\sin(30)} = 2$ and $\frac{1}{\sin(-30)} = -2$
* $\cos(30) = \frac{\sqrt{3}}{2}$ and $\cos(-30) = \frac{\sqrt{3}}{2}$

Therefore, $\sin x$ and $\operatorname{cosec} x$ are odd functions while $\cos x$ is an even function (where $f(-x) = f(x)$).

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function is one-to-one iff (if and only if)

$$f(x_1) = f(x_2) \rightarrow x_1 = x_2$$

$$\text{or } f(x_1) \neq f(x_2) \rightarrow x_1 \neq x_2$$

(15) $f(x) = \frac{f(x+h) - f(x)}{h}$ is the derivative of a function.

{16} $P(x_0, y_0)$ & $Q(x_1, y_1)$.

$\frac{dy}{dx} = \frac{y_1 - y_0}{x_1 - x_0}$, but $\tan\theta = \frac{dy}{dx}$.

$\tan^2\theta + 1 = \sec^2\theta$.

$\left(\frac{y_1 - y_0}{x_1 - x_0}\right)^2 + 1 = \sec^2\theta$.

$\frac{(y_1 - y_0)^2 + (x_1 - x_0)^2}{(x_1 - x_0)^2} = \sec^2\theta$

but $(y_1 - y_0)^2 + (x_1 - x_0)^2 = 1$

$\frac{1}{(x_1 - x_0)^{\cancel{2}}} = \sec\cancel{\theta}^{\cancel{2}}$

$\sec\theta = \left(\frac{1}{x_1 - x_0}\right)$

$\sec\theta = \frac{1}{x_1 - x_0}$.

{17} $\frac{dy}{dx} = fx$.

$\frac{y_1 - y_0}{x_1 - x_0} = \frac{dy}{dx} = fx$.

$\therefore$ Solution of points are (x_0, y_0) & (x_1, y_1)

(18) $f(x) = \frac{3x - 5}{2x^2 - x - 3}$

NOTE: The points at which a function is continuous refer to the domain of the function where all points outside the domain are not included.

let denominator $= 0$

$\Rightarrow 2x^2 - x - 3 = 0$

$2x^2 + 2x - 3x - 3 = 0$

$2x(x+1) - 3(x+1) = 0$

$(2x - 3)(x+1) = 0$

$2x - 3 = 0 \Rightarrow x = \frac{3}{2}$ or $x + 1 = 0 \Rightarrow x = -1$

$\therefore$ The domain of the function is the continuous points of the function for all real numbers apart from $\frac{3}{2}$ and -1

Answer: $\{x : x \in \mathbb{R} \mid x \neq \frac{3}{2}$ or $x = -1\}$ or $(-\infty, -1) \cup [0, \frac{3}{2}) \cup (\frac{3}{2}, +\infty)$

(22) $f(x) = \frac{x}{x^4 - 16}$

let $x^4 - 16 = 0$

$x^4 = 16$

$x = \sqrt{16} = 2$ (The function is continuous at all points except 2).

$\Rightarrow \{x \in \mathbb{R} \mid x \neq 2\}$ or $(-\infty, 2) \cup (2, +\infty)$.

(11) $y = 5x^3e^x + \ln x$

$y' \Rightarrow 5x^3e^x$ (using product rule) and y' of $\ln x = \frac{1}{x}$

$= \left[5x^3 \cdot e^x + 15x^2 \cdot e^x\right] + \frac{1}{x}$

$= \underline{5x^3e^x + 15x^2e^x + \frac{1}{x}}$ or $5x^3e^x + 15x^2e^x + x^{-1}$

(12) $x^3 - xy + y^2 = 4 \quad (0, -2)$

$y' = 3x^2 - (xy' + y) + 2yy' = 0$

$3x^2 - xy' - y + 2yy' = 0$

$-xy' + 2yy' = -3x^2 + y$

$\frac{y'\cancel{(-x+2y)}}{\cancel{(2y-x)}} = \frac{-3x^2 + y}{(2y-x)}$

$y' = \frac{y - 3x^2}{2y - x}$

at $(0, -2)$

$\Rightarrow y' = \frac{-2 - (3 \times 0^2)}{2(-2) - 0}$

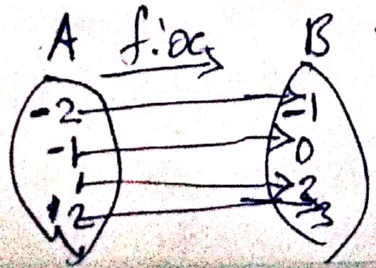
$= \frac{-2}{-4} = \frac{1}{2}$

{13}

$f(x) = x + 1$ is onto?

to integer $x = \{-2, -1, 1, 2\}$

$f(-2) = -1, f(-1) = 0, f(1) = 2, f(2) = 3.$



Since element A have unique element in B

∴ therefor is one to one

not Onto. false.

if $\lim_{x \to a} f(x) = f(a)$ and $\lim_{x \to b} f(x) = f(b)$, the f is continuous from b to a.

(24) Find $h^{-1}(x)$ if $h(x) = \frac{3x}{2x+1}$

Let $y = h(x)$

$\Rightarrow y = \frac{3x}{2x+1}$

$x = \frac{3y}{2y+1}$

$x(2y+1) = 3y$

$2xy + x = 3y$

$2xy - 3y = -x$

$\frac{y\cancel{(2x-3)}}{\cancel{(2x-3)}} = \frac{-x}{(2x-3)}$

$y = -\frac{x}{2x-3}$

$\therefore h^{-1}(x) = \frac{-x}{2x-3}$ or $\frac{x}{3-2x}$

(25) A function is said to be even if:
$f(-x) = f(x)$ and * odd if $f(-x) = -f(x)$.

{27} $f(x) = x^3 + 2$.

$y = x^3 + 2$.

$y - 2 = x^3$.

$x = \sqrt[3]{y-2}$.

$f^{-1}(x) = \sqrt[3]{x-2}$

(28) if $f(x) = x-1$ and $g(x) = \sqrt{x}$, find the domain of $g \circ f$.

$$g \circ f = \sqrt{x-1}$$

$\Rightarrow D = [1, +\infty)$ (This is for interval notation)

or

$\{x \in \mathbb{R} \mid 1 \leq x < +\infty\}$ (This is the set-builder notation).

(29) From Q28 above, $g \circ f = \sqrt{x-1}$

(30) If it Continuous at $a \leq x \leq b$ or $[a\ b]$.

(31) $\int dx = x + c$ because $\frac{dy}{dx}$ of $x = 1$ which is also same as $\int 1\,dx = \int dx$

$$\int dx = \int x^0 dx = \frac{x^{0+1}}{0+1} = x + c.$$

(33) Composition of $f \circ g$ is $f \circ g(x) = f(g(x))$

(34) If $f(x) = x$, $g(x) = x$ $f \circ g(x) = g \circ f(x) = x$

(35) / (32) $f x = \frac{f(x + \Delta x) - f x}{\Delta x}$.
at $\Delta x \to 0$

(36) $\frac{d(10x-7)}{dx} = 10$

$$f(x) = \frac{f(x + (x - x_0)) - f(x)}{x - x_0}.$$
at $x - x_0 \to 0$

$$f(x) = \frac{f(2x - x_0) - f(x)}{x - x_0}$$
at $x - x_0 \to 0$

(38) $\int x^3 - xy + y^2 = 4$

(37) $\int (x^3 - xy + y^2)\,dx = \frac{x^4}{4} - \frac{yx^2}{2} + y^2 x$

$$\frac{1}{4}x^4 - \frac{1}{2}yx^2 + xy^2 + k.$$

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Same as (11)

(39) $y = 5x^3 e^x$

$\frac{dy}{dx} = 5x^3 e^x + 15x^2 e^x$

~~$5x^2 e^x$~~

$5x^2 e^x (x + 3e^x)$

(40) $f(x) = 2x+1$ and $g(x) = 3x+4$

$f \circ g \Rightarrow f(g(x))$

$\Rightarrow 2(3x+4)+1$

$6x + 8 + 1 \Rightarrow 6x+9$

or $3(2x+3)$

(41) $u+v = uv' + vu' \Rightarrow y'$ (derivative).

(42) $f(x) = 8-2x$, find $f(-1)^{-1}$

$y = 8-2x$

$x = 8-2y$

$\frac{2y}{2} = \frac{8-x}{2}$

$y = \frac{8-x}{2}$ (inverse $\Rightarrow f^{-1}(x)$)

at $x = -1$

$\Rightarrow \frac{8-(-1)}{2} = \frac{8+1}{2} = \frac{9}{2}$ or $4\frac{1}{2}$ or 4.5

(43) If $f(a) = f(b)$ implies that $a=b$ is a/an one-to-one/injective function.

(44) $y = 5x^3 + 2x$

$y' = 15x^2 + 2$

(45) $\lim_{x \to 5} 10 = 10$

because $\lim_{x \to a} C = C$

~~46~~

(46) $y = -\frac{1}{3}(x^7 + 2x - 9)$

$\Rightarrow -\frac{1}{3}(7x^6 + 2)$

(47) All options are odd functions except (c)

$\Rightarrow -3x^2$

Let $x = 1$ and -1

$-3(1^2)$ and $-3(-1^2)$

-3

$\Rightarrow -3$

Since all values of x have no effect on the coefficient, it implies the function is even.

(48) Same as Q2.

(49)

(50) $y = -4x + 16$

equation of a line is given by $y = mx + c$

$y = mx + c$ ——— (1)

$y = -4x + 6$ ——— (2)

Compare (1) and (2)

$\Rightarrow m = -4$ (where m is the slope/gradient).

$\frac{d}{dx} = f(x) \Rightarrow$ Derivative.

(54) $y = 4x^3 y^3 - y = x$ question can not be like this

$y' = 12x$

(55) $\int f(x)\,dx$, find its anti-derivative

$\Rightarrow \int f^{-1}(x)\,dx + C$

(56) $\lim_{x \to -2} \left(2x - \sqrt{4x^2 + x}\right)$

direct substitution

$= 2(-2) - \sqrt{4(-2^2) + (-2)}$

$= -4 - \sqrt{16 + (-2)} \Rightarrow -4 - \sqrt{14}$

{19} $\int f(x)\,dx = f(x) + c$.
find $f^{-1}(x) = f(x)$.

$\therefore \frac{dy}{dx} = f(x)$.

Such that $dy = fx\,dx$.

$y = \int f(x)\,dx$.

but $y = fx$ $fx = \int fx\,dx$

(20) The derivative of the function f is a function, f^{-1} is define by Anti derivative of f.

{21} $y = x^2 \cos x$

$\frac{dy}{dx} = -x^2 \sin x + 2x\cos x$.

$\frac{dy}{dx} = x(2\cos x - x\sin x)$.